UDC 681.5.015.52

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DETERMINISTIC IDENTIFICATION METHODS FOR NONLINEAR DYNAMICAL SYSTEMS BASED ON THE VOLTERRA MODEL

Abstract. *The paper solves an important scientific and practical problem, which is to improve the accuracy and computational stability of the methods of deterministic identification of nonlinear dynamic systems in the form of Volterra model based on experimental data of observations "input-output" taking. On the base of theoretical and experimental studies created effective instrumental algorithmic and software tools for estimating Volterra kernels in the time domain Into account measurement errors. Results of the further development of methods of deterministic identification of nonlinear dynamic systems based on Volterra models using irregular pulse sequences show. The methods are based on the use of the Tikhonov regularization procedure. The amplitude of test impulses is used as a regularization parameter. In the identification, procedure applies wavelet filtering for smooth the estimates of the Volterra kernels apply. This gives increase the accuracy and noise immunity of identification methods. The approximation method of identification of the nonlinear dynamic systems based on Volterra models is improved. Method is consists in the choice of amplitudes of test signals and of coefficients scaling of the partial components of responses a nonlinear system in procedure of processing of signals-responses. The improvement is reduced to minimizing the methodological error in the allocation of partial components from the response of the identification object and allows obtaining more accurate estimates of Volterra nuclei. To improve the computational stability of the developed identification algorithms and for noise reduction in the obtained estimates of multidimensional Volterra kernels the wavelet filtration is used. This allows obtaining smoothed solutions and decreases error of the identification by 1,5-2,5 times. A new robust method of deterministic identification of nonlinear dynamic systems based on Volterra models in the time domain is developed. In contrast to the interpolation method, where finite difference formulas with a predetermined number of experimental studies of the object of identification are used for numerical differentiation. It is proposed to solve the corresponding Volterra integral equations of the first kind, for the numerical implementation of which an unlimited number of experiments can be used. This makes it possible to increase the accuracy of the calculation of derivatives, and consequently, the accuracy of identification. Software tools on the system Matlab platform have been developed to implement the developed computational algorithms for deterministic identification of nonlinear dynamic systems in the form of Volterra kernels.*

Keywords: *nonlinear dynamical systems; identification; Volterra model; Volterra kernels; ill-posed problem; Tikhonov regularization; wavelet transformation*

Introduction

Mathematical modeling methods and experiment [1-5] are the main research means of complex nonlinear dynamical systems (NDS). For exposition of NDS an appliance of Volterra integro-power series is often used [6-13]. Nonlinear and dynamical properties of the system are fully characterized by a sequence of multidimensional weight functions – Volterra kernels. NDS identification problem – creation of model as Volterra series – is based on identification of Volterra kernels using experimental "input-output" system investigation data [14-18].

Identification itself is a reverse task so during solving such severe calculation problems occur due to inconsistent problem definition. The occurred results appear to be unstable to input data errors due to changes in replies of the identified NDS [19-20].
In addition, in case of the use of models as Volterra serie some NDS for partial component $y_n(t)$, which correspond to different parts of Volterra series, since $y(t)$, total reply to the input signal $x(t)$ [15]. Thus, it is required to use specialized difficulties rise in separation of the $y(t)$ reply of examined approaches for extraction of partial component out of NDS replies. Different methods of such decomposition based on compensation [21-22], approximation [23-25] and interpolation [26-28] approaches are proposed. NDS deterministic identification method with the use of test no regular impulse sequences is shown. The advantage of proposed methods in comparison to statistic identification [29] based ones lies in simplified experimental data processing and implementation of test signals. Though deterministic identification results a greatly impacted by measurement error which narrows its application in real experiments. To improve the computational stability of the algorithms identification the method of A. N. Tikhonov regularization of ill-posed problems [30-31], is used and demising procedures in the estimates of multidimensional Volterra kernels based on the wavelet transform [32-34].

The main goal is to examine errors, which appear while determined identification is applied to the NDS with unknown structure in the real life experiment, comparison analysis of its preciseness, and noise proof efficiency.

1. Volterra Models

In the general case the "input-output" relationship for a nonlinear dynamical system can be represented in terms of the Volterra series as

$$y[x(t)]=\sum_{n=1}^{\infty} y_n(t)= \\ =\sum_{n=1}^{\infty}\int_0^t \ldots \int_0^t w_n(\tau_1,...,\tau_n)\prod_{i=1}^{n} x(t-\tau_i)d\tau_i, \quad (1)$$

where $x(t)$ and $y[x(t)]$ are the input and output signals, respectively, $w_n(\tau_1,...,\tau_n)$ is the Volterra kernel of the n-th order and $y_n(t)$ stands for the n-th partial component of the system response.

Commonly, the Volterra series are replaced by a polynomial, with only taking several first terms of series (1) into consideration. Then the identification procedure consists in extracting the partial components with subsequent determination of Volterra kernels $w_n(\tau_1,...,\tau_n)$. They are a nonparametric model of the input-output system under study. The output function of the model $\hat{y}(t)$ approximately describes the system output $y[x(t)]=y(t)$ for a given input signal $x(t)$.

The block diagram of the Volterra model in the time domain is shown in Fig. 1.

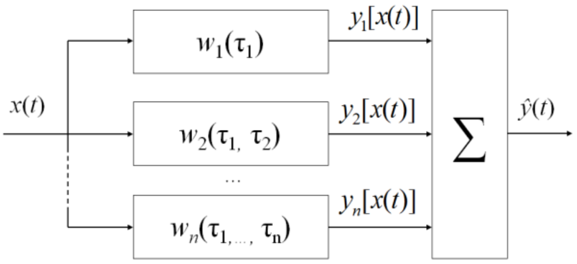


Fig. 1. Volterra model in the time domain

For descriptions of NDS with multiple input and multiple output a multivariate Volterra series is used:

$$y_j(t)=\sum_{i_1=1}^{\nu}\int_0^t w^j_{i_1}(\tau)x_{i_1}(t-\tau)d\tau+ \\ +\sum_{i_1,i_2=1}^{\nu}\int_0^t\int_0^t w^j_{i_1i_2}(\tau_1,\tau_2)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)d\tau_1 d\tau_2+ \\ +\sum_{i_1,i_2,i_3=1}^{\nu}\int_0^t\int_0^t\int_0^t w^j_{i_1i_2i_3}(\tau_1,\tau_2,\tau_3)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)\times \\ \times x_{i_3}(t-\tau_3)d\tau_1 d\tau_2 d\tau_3\ldots,\quad j=\overline{1,\mu}, \quad (2)$$

where $y_j(t)$ – responses of the NDS on j-th output in case zero start values; $x_1(t),...,x_\nu(t)$ – input signals; $w^j_{i_1i_2\ldots i_n}(\tau_1,...,\tau_n)$ – Volterra kernel of n-th order based on $i_1,...,i_n$ input and j-th output, function is symmetric for real arguments $\tau_1,\ldots,\tau_n$; ν, μ – amount of input and output NDS outlets correspondingly.

In real live Volterra serie changed to polynomial so only, few first elements are being taken. The model creation of NDS as Volterra serie lies in choice of test signals $x(t)$ and algorithm development which allows to show partial component $y_n[x(t)]$ based on measured results $y(t)$ and estimation Volterra kernels where $w_n(\tau_1,\ldots,\tau_n)$, $n=1,2,\ldots,\infty$.

Wide usage of models as Volterra series for identification and modeling of NDS, usage of modeling results in a huge variety of appliances, is explained by its major preferences such as invariativity to input signal type, which means that a problem could be solved for both determined and random input signal; explicit dependency for input and output variables; versatility – a possibility to examine nonlinear continuous in time and nonlinear impulse systems, steady-state and rheo nomic systems, with lumped and distributed parameters, stochastic systems as well as systems with multiple input and multiple output outlets. Grants a possibility of examination in analytic and computable applications; simultaneous and compact use of nonlinear and inertial NDS properties; interpolation of linear systems as subclass of nonlinear systems, which allows to use time-based and spectral-based methods designed for linear systems for nonlinear systems as well.

2. Identification Methods

2.1. Compensation Method. The modified compensation method for identification NDS in the form Volterra kernels in time domain is based on testing the system under study with using irregular impulse sequences with varying parameters: amplitude, duration of test pulses and intervals between them [21-22].

The model of the test signal in the form of an irregular sequence containing no more than n rectangular pulses of duration $\Delta\tau$ with different amplitudes a_k acting at the time τ_k (k=1, 2, …, n), has the form

$$x(t)=\sum_{k=1}^{n}\theta_k S_k\delta(t-\tau_k),\quad \tau_k\in[0,t], \quad (3)$$

where $S_k=a_k\Delta\tau$ – the area of the k-th pulse in the test sequence; $\delta(t-\tau_k)$ – the Dirac delta function; t – the current time; θ_k – the parameters that determines the number of pulses in the sequence and the intervals between them; if $\theta_k=1$, then at the time τ_k in the sequence of the pulse is; if $\theta_k=0$ – none.

Let the amplitude of the test pulses $a_1,..., a_n$. For sufficiently small values of the duration $\Delta\tau$ and amplitude a_k (k=1,...,n) pulses, the following statements are true. Their proof using the method of mathematical induction is given in the Appendix.

The following statements that define the computational algorithm of the compensation identification method are valid.

Statement 1. Let the test signals be irregular pulse sequences of various lengths, each of which consists of no more than n pulses acting at times $\tau_1,..,\tau_n$. Then for the NDS with one input and one output, the estimate of the cross section of the Volterra kernel of the *n*-th order is:

$$\hat{w}_n(t-\tau_1,...,t-\tau_n)=\left(n!\prod_{k=1}^{n}S_k\right)^{-1}\times \\ \times\sum_{\theta_1,...,\theta_n=0}^{1}(-1)^{n+\sum_{k=1}^{n}\theta_k}y(t,\theta_1,...,\theta_n), \quad (4)$$

where $y(t,\theta_1,...,\theta_n)$ – is response of NDS, which is measured at t moment, under the operation of modulated delta-impulses with S_k square in proportion to time point of $\tau_1,\ldots,\tau_n$. If $\theta_k=1$, then there is impulse in NDS input at τ_k time point, but if $\theta_k=0$, there is none.

Statement 2. There occur such proportion for the definition Volterra kernel of *n* order NDS with ν input and μ output:

$$\hat{w}^{j}_{i_1,...,i_n}(t-\tau_1,...,t-\tau_n)=\left(n!\prod_{k=1}^{n}S_k\right)^{-1}\times \\ \times\sum_{\theta_1^{i_1},...\theta_n^{i_n}=0}^{1}(-1)^{n+\sum_{k=1}^{n}\theta_k^{i_k}}y_j(t,\theta_1^{i_1},...,\theta_n^{i_n}), \quad j=\overline{1,\mu}, \quad (5)$$

where $w^{j}_{i_1,...,i_n}(t-\tau_1,...,t-\tau_n)$ – is estimation of n order Volterra kernels, which is the result of data processing of experiment; $y_j(t,\theta_1^{i_1},...,\theta_n^{i_n})$ – is response of the object, which is measured at *j*-output at t point time under the operation of $i_1,...,i_n$ modulated delta-impulses with S square in proportion to time point of $t_1,\ldots t_n$,. If $\theta_k^{i_k}=1$, then there is impulse in i_k input NDS at τ_k time point, but if $\theta_k^{i_k}=0$, there is none.

For example, to determine the of Volterra kernel of the NDS second order is tested by single pulses, which are fed at the moments of time τ_1 and τ_2:

$$x_1(t)=S_1\delta(t-\tau_1) \text{ and } x_2(t)=S_2\delta(t-\tau_2). \quad (6)$$

The corresponding responses and are measured. Then, two pulses are fed to the NDS input

$$x(t)=S_1\delta(t-\tau_1)+S_2\delta(t-\tau_2), \quad (7)$$

and from the resulting response is subtracted responses to single pulses

$$y(t,1,1)-y(t,1,0)-y(t,0,1)= \\ =2!S_1S_2\hat{w}_2(t-\tau_1,t-\tau_2). \quad (8)$$

From (4), after normalization, it follows:

$$\hat{w}_2(t-\tau_1,t-\tau_2)=\frac{1}{2!S_1S_2}[y(t,1,1)- \\ -y(t,1,0)-y(t,0,1)]. \quad (9)$$

At fixed values τ_1 and τ_2 estimation Volterra kernel of the second order $\hat{w}_2(t-\tau_1,t-\tau_2)$ a function of the variable t – section of the surface by a plane passing at an angle of 45^0 to the axes t_1 and t_2 and shifted along the axis t_1 by a value $\tau_0=\tau_2-\tau_1$. By changing the value τ_0, get various sections $\hat{w}_2(t,t-\tau_0)$, which can restore the entire surface $\hat{w}_2(t_1,t_2)$.

A schematic representation of the procedure for identification Volterra kernel second order of the NDS with one input and one output and with two inputs and one output show in Fig. 3 and 4 respectively.

A schematic representation of the procedure for identification Volterra kernel third order of the NDS with two inputs and one output $\hat{w}_{112}(t,t-\tau_1,t-\tau_2)$ show in Fig. 3. As a result of such operations, we have:

$$y(t,1_1,1_1,1_2)-(y(t,1_1,0,0)++y(t,0,1_1,0)+ \\ +y(t,0,0,1_2))+y(t,1_1,1_1,0)+y(t,1_1,0,1_2)+ \\ +y(t,0,1_1,1_2)=3!S^3\hat{w}_{112}(t,t-\tau_1,t-\tau_2). \quad (10)$$

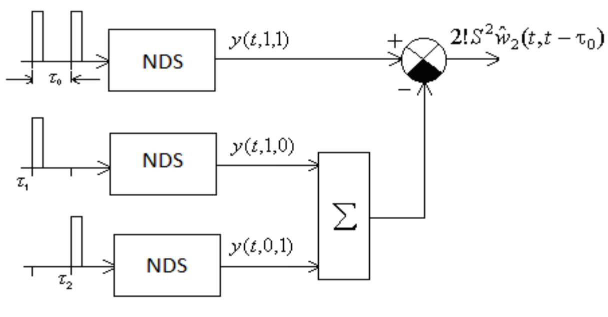


Fig. 2. Procedure identification of the Volterra kernel second order

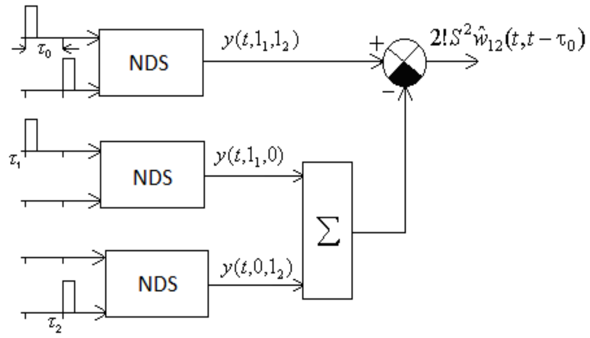


Fig. 3. Procedure identification of the Volterra kernel second order at different inputs

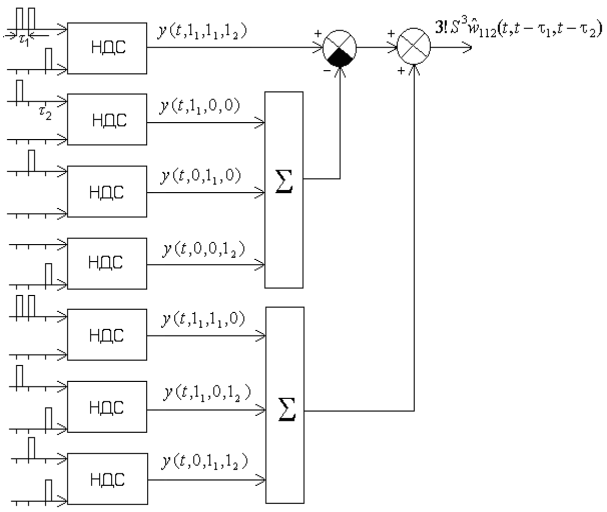


Fig. 4. Procedure identification of the Volterra kernel third order at different inputs

As a result of processing the responses $y(t,\theta_1,...,\theta_m)$ and $y_j(t,\theta_1^{i_1},...,\theta_n^{i_n})$ of the NDS in accordance with (4) and (5), the approximate values of the cross sections of the Volterra kernels are obtained. The accuracy of identification depends on the choice of the area of the test pulses S_k, i.e. the duration and amplitude of the pulses.

With decreasing the pulse amplitude of the test sequence, its optimal value corresponding to the minimum error of the experimental determination of the Volterra kernels based on the compensation method of identification is found. Since in the conditions of the real experiment the measurements of the NDS responses are carried out with some instrumental error, the relative measurement error (a random error) will increase when the amplitude of the test pulses decreases. The instability of the computational algorithms of deterministic identification (4) and (5) to the errors of the initial data — measurements of pulse responses, especially strongly affects the determination of high order Volterra kernels. Practical implementation of the algorithms is possible only in conditions of relatively small noise levels in the measurement of NDS responses. To improve the accuracy of the identification method, procedures can also be used to suppress the response components of all even and all odd orders.

If it is known that NDS is described by a functional polynomial of power N, then when determining the n-th order by the compensation method, the methodical error will be zero. The determination of Volterra kernels below the N-th order is made by sequentially lowering the order of the NDS model. In this case, the components of the response from the Volterra kernel of the higher orders model are subtracted from the output signal of the system.

2.2. Approximation Method. The approxima-tion method identification in domain time it is based on the allocation of the n-th partial component in the NDS response by constructing linear combinations of responses to test signals with different amplitudes [23-24].

The amplitudes of test signals which were proposed for usage of approximated method of identification are not optimal and do not provide with minimum error of multidimensional identification Volterra kernels system. The following affirmations are correct.

Statement 3. Let at system input test signals are given successively $a_1x(t)$, $a_2x(t)$,…,$a_Nx(t)$ (N – is approximation model order, a_1, a_2,…,a_N – different real numbers, which satisfy the term $0<|a_j|\le 1$ for $\forall j=1,2,...,N$; $x(t)$ – arbitrary function). Then linear NDS combination of responses with coefficient c_j amount to n partial component of NDS response in case input signal $x(t)$ with error due to higher orders partial components, $n>N$:

$$\sum_{j=1}^{N} c_j y[a_j x(t)] = y_n[x(t)] + \sum_{j=1}^{N} c_j \sum_{n=N+1}^{\infty} y_n\left[a_j x(t)\right], \qquad (11)$$

where c_j — real coefficients such that

$$\begin{bmatrix} a_1 & a_2 & \dots & a_N \\ a_1^2 & a_2^2 & \dots & a_N^2 \\ \dots & \dots & \dots & \dots \\ a_1^N & a_2^N & \dots & a_N^N \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_N \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix}, \qquad (12)$$

and $b_k = 1$ at $k = n$ and $b_k = 0$ at $k \ne n$, $k = \overline{1,N}$; $\forall n \in \{1,2,\dots,N\}$.

Let input test signals $x(t)$ present themselves in irregular sequences impulses of different length. Each sequences consists of not more than of n delta impulses with area of $S=\Delta\tau\Delta x$ ($\Delta\tau$ – duration, Δx – the amplitude of the rectangular impulses), which function at $t_1 \dots t_n$ time point. Then for NDS with one input and output the estimation of diagonal section of Volterra kernel n-th order is:

$$\hat{w}_n(t-\tau_1,...,t-\tau_n) = = \frac{(-1)^n}{n!(\Delta\tau)^n} \sum_{\theta_1,...,\theta_n=0}^{1} (-1)^{\sum_{k=1}^{n}\theta_k} \hat{y}_n(t,\theta_1,...,\theta_n), \qquad (13)$$

where $\hat{y}_n(t,\theta_1,...,\theta_n)$ – is estimation of n-th partial component of the NDS response at t time point, which was obtained in the result of data processing of experiments on bases of (11).

The estimation of the diagonal section Volterra kernel n-th order

$$\hat{w}_n(t,...,t) = \frac{\hat{y}_n(t)}{(\Delta\tau)^n} \qquad (14)$$

where $\hat{y}_n(t)$ – is estimation of n-th partial component of the NDS response at single impulse at t time point, which was obtained as the result of data processing of experiments on bases of (11).

Statement 4. To minimize the influence of the Volterra serie balance on the error in the allocation of the partial component NDS response (12), it is necessary to provide a minimum of the sum of the modules of the coefficients c_j (j=1,2,...,N), which are determined from a system of equations (12)

$$\varepsilon = \sum_{j=1}^{N} |c_j| = \sum_{j=1}^{N} |a_{jk}^{-1}| \to \min, \qquad (15)$$

where a_{jk}^{-1} is elements of the inverse matrix

$$\mathrm{A} = \begin{bmatrix} a_1 & a_2 & \dots & a_N \\ a_1^2 & a_2^2 & \dots & a_N^2 \\ \dots & \dots & \dots & \dots \\ a_1^N & a_2^N & \dots & a_N^N \end{bmatrix}. \qquad (16)$$

According to (15) the task of providing methodical minimum error reduces to discovering of local functional minimum of ensemble variables by means of usage the approximated identification method. In other words, it is sum of rate modulus in linear combination of responses. By means of complete searching procedures of different amplitude value, appropriate rate is discovered and each time the system of linear algebraic equations (12) is solved.

After calculating the form, there should be got optimal amplitude value. In [25] it is illustrated that the decreasing of methodical error of identification method could be reached by two means; by selection of rather low amplitude of test signals with pre-assigned approximated order or by fixed amplitude with increasing of approximated model order.

2.3. Interpolation Method. There was proposed interpolation method of NDS identification on base of PB in [26-28], where for splitting of NDS response at PS $\hat{y}_n(t)$ is used and it is multiple differencing of output signal according to parameter of amplitude test signals. The following affirmations are proved.

Statement 5. Let at input of system test signal of $ax(t)$ kind is given, where $x(t)$ – is arbitrary function and a – is the coefficients of scale (amplitude of signal), where $0<|a|\le 1$. Then for marking out partial component of n order $\hat{y}_n(t)$ from measured NDS response $y[ax(t)]$ it is necessary to find n partial component of response according to the amplitude a where a=0:

$$n!\underbrace{\int_0^t ... \int_0^t}_{n\ times} w_n(\tau_1,...,\tau_n)\prod_{k=1}^{n} x(t-\tau_k)d\tau_k = \qquad (17)$$
$$= y_a^{(n)}[a\,x(t)]_{a=0}$$

While using test irregular impulse series with $\Delta\tau$ duration, could be discovered diagonal and sub-diagonal Volterra kernel section on the basis of (15) and (14) forms accordingly. It is taken into consideration that calculation of $\hat{y}_n(t)$ and $\hat{y}_n(t,\theta_1,...,\theta_n)$ is made by means of (17) procedure.

Partial component should be substituted by form of finite difference for calculation of (17). Differentiation of function, which was set in discrete spots, could be accomplished by means of numerical computing after preliminary smoothing of measured results.

Various formulas for the numerical differentiation known. They vary from each other by means of error.

Let's use universal reception which allows to substitute a derivative of any n order for differential ratio so that the error from such replacement for function $y(a)$ was any beforehand set order of p approximation concerning a step of $h=\Delta a$ of computational mesh on amplitude [35]. Method of undetermined coefficients for the equality:

$$\frac{d^n y(a)}{da^n} = \frac{1}{h^n}\sum_{r=-r_1}^{r_2} c_r y(a+rh)+O(h^p), \qquad (18)$$

let's pick up coefficients c_r not depending on h, $r=-r_1,-r_1+1,...,r_2$, so that equality (18) was fair. Limit of sum $r_1 \ge 0$ и $r_2 \ge 0$ could be arbitrary, but so that the differential relation $h^{-n}\sum c_r y(a+rh)$ of r_1+r_2 order satisfies to inequality $r_1+r_2 \ge n+p-1$.

For definition of c_r it is necessary to solve the following set of equations

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -r_1 & -r_1+1 & \cdots & r_2 \\ \cdots & \cdots & \cdots & \cdots \\ (-r_1)^{n-1} & (-r_1+1)^{n-1} & \cdots & r_2^{n-1} \\ (-r_1)^{n} & (-r_1+1)^{n} & \cdots & r_2^{n} \\ (-r_1)^{n+1} & (-r_1+1)^{n+1} & \cdots & r_2^{n+1} \\ \cdots & \cdots & \cdots & \cdots \\ (-r_1)^{n+p-1} & (-r_1+1)^{n+p-1} & \cdots & r_2^{n+p-1} \end{bmatrix} \cdot \begin{bmatrix} c_{-r_1} \\ c_{-r_1+1} \\ \cdots \\ c_{r_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ n! \\ 0 \\ \cdots \\ 0 \end{bmatrix}. \qquad (19)$$

When $r_1+r_2=n+p-1$ then inscribed in $n+p$ equality forms linear system concerning the same number of c_r unknown. The determiner of this system is Vandermond's determiner and differs from zero. Thus, there is the only one set of n coefficients, satisfying the system (19). When $r_1 + r_2 \geq k + p$ then obviously quite a number of such c_r coefficients systems exist.

On the basis of (18) in [28] the formulas of derivative calculation of the first, second, third and fourth orders are received at *a*=0 with use of the central and right differences for equidistant assembly.

In paper, formulas for numerical differentiation with use of the central differences for equidistant assembly are used. For definition of first Volterra kernels order the first derivative is calculated at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$ respectively on formulas

$$y_0' = \frac{1}{2h}(-y_{-1} + y_1),$$
$$y_0' = \frac{1}{12h}(y_{-2} - 8y_{-1} + 8y_1 - y_2). \quad (20)$$

For definition of second Volterra kernels order the second derivative is calculated at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$ respectively on formulas

$$y_0'' = \frac{1}{h^2}(y_{-1} + y_1),$$
$$y_0'' = \frac{1}{12h^2}(-y_{-2} + 16y_{-1} + 16y_1 - y_2). \quad (21)$$

For definition of third Volterra, kernels order the third derivative is calculated at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$ respectively on formulas

$$y_0''' = \frac{1}{2h^3}(-y_{-2} + 2y_{-1} - 2y_1 + y_2),$$
$$y_0''' = \frac{1}{8h^3}(y_{-3} - 8y_{-2} + 13y_{-1} - 13y_1 + 8y_2 - y_3). \quad (22)$$

In formulas (20) – (22) the notation is entered:

$y_0' = y'(0),\ y_0'' = y''(0),\ y_0''' = y'''(0);\ y_r = y(rh),\ r = 0, \pm 1, \pm 2; \pm 3.$

The corresponding partial component is found by the formula (18). Sections of Volterra kernels by the diagonal and the sub diagonal are calculated on the basis of expressions (14) and (13).

2.4. Robust Method. Proposed robust method of deterministic identification of the NDS based on Volterra model in the time domain [36-39]. Irregular pulse sequences are used as test signals. The stability of the computational process of the identification procedure is ensured by using the method of A. N. Tikhonov regularization of ill-posed problems [30].

The problem of finding the derivative of *n*-th order z(a) from the function y(a), for which $y(0)=y'(0)=...=y^{(n-1)}(0)=0$, reduces to solving the Volterra integral equation of the first kind [30] with respect to $z(\xi)$:

$$\int_0^a \frac{1}{(n-1)!}(a-\xi)^{n-1} z(\xi) d\xi = y(a) \quad (23)$$

This problem is characterized by the lack of stability of the solution to small changes in the right side of the equation $y(a)$. To find the approximate solution $z(\xi)$ of equation (23), which is resistant to the errors of the initial data, the method of regularization of ill-posed problems is applied [7; 15].

The problem of estimation diagonal section of Volterra kernel *n*-th order is the solution of the integrated equation Volterra of the first sort (23). For realization of an algorithm of identification (13) and (14) we will pass to a discrete analog of a problem of finding of regularized approximate solutions of the equation (17). Let us measure NDS responses on a set of trial impulse signals with amplitude of impulses change discretely on $0 < a \leq a_{\max}$ with Δa step. Then each data set for the specified point-in-time value *t* from of the received set of responses $y(a_i, t, \theta_1, ..., \theta_n) = y(i\Delta a)$, where $a_i = i\Delta a$, $i = 1, 2, ..., L$ (*L* – the number of levels of sampling on amplitude *a*) subjected to the operation of *n*-fold numerical differentiation by *a*. Such a procedure comes down to the solution of the system of linear algebraic equations:

$$\sum_{j=0}^{i} \frac{\Delta a}{(n-1)!}(a_i - \xi_j)^{n-1} z(\xi_j) = u(a_i), \quad (24)$$

where $\xi_j = j\Delta a$, $\Delta a = a_{\max}/L,\ u(a_i) = \sigma(a_i) y(a_i)$; $\sigma(a)$ – some function for which conditions are satisfied:

$$\sigma(0) = \sigma'(0) = \ldots = \sigma^{(n)}(0) = 0,$$
$$\sigma(a_{\max}/2) = 1/2,\ \sigma(a_{\max}) = 1, \quad (25)$$
$$\sigma'(a_{\max}) = \ldots = \sigma^{(n)}(a_{\max}) = 0.$$

As function, $\sigma(a)$ it is possible to choose, for example, sigmoid function

$$\sigma(a) = \frac{1}{1 + \exp\lambda(-a + a_{\max}/2)}. \quad (26)$$

The system of the equations (24) can be written down in a vector-matrix form

$$\mathrm{A}_n \bar{z} = \bar{u}, \quad (27)$$

where

$$\mathrm{A}_n = \left[\alpha_{ij}^{(n)}\right]_{i=\overline{1,L};\ j=\overline{0,(L-1)}},$$

$$\alpha_{ij} = k_n (i-j)^{n-1}, \text{ for } j = \overline{1,(i-1)};$$

$$\alpha_{i0}^{(n)} = \frac{k_n i^{n-1}}{2}, \text{ for } j=0;$$

$$\alpha_{ij} = 0, \text{ for } j>i;$$

$$k_n = \frac{(\Delta a)^n}{(n-1)!};$$

$\bar{z} = (z_1,\ldots,z_L)'$, $z_i = z(i\Delta a)$;

$\bar{u} = (u_1,\ldots,u_L)'$, $u_j = u(j\Delta a) = \sigma(j\Delta a)y(j\Delta a)$.

The matrix A_n for n=1, 2 and 3 are the form, respectively

$$A_1 = \Delta a\begin{bmatrix} 1/2 & 0 & 0\ldots 0 \\ 1/2 & 1 & 0\ldots 0 \\ 1/2 & 1 & 1\ldots 0 \\ \ldots & \ldots & \ldots \\ 1/2 & 1 & 1\ldots 1 \end{bmatrix} \quad (28)$$

$$A_2 = (\Delta a)^2 \times \begin{bmatrix} 1/2 & 0 & 0 & \ldots & 0 \\ 1 & 1 & 0 & \ldots & 0 \\ 3/2 & 2 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ L/2 & (L-1) & (L-2) & \ldots & 1 \end{bmatrix} \quad (29)$$

$$A_3 = \frac{(\Delta a)^3}{2} \times \begin{bmatrix} 1/2 & 0 & 0 & \ldots & 0 \\ 2 & 1 & 0 & \ldots & 0 \\ 9/2 & 4 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ L^2/2 & (L-1)^2 & (L-2)^2 & \ldots & 1 \end{bmatrix}. \quad (30)$$

The required solution of $z(i\Delta a)$ is found at i=0 (z_0). Then, we receive

$$z_0 = u'(0) = y'(0)\sigma(0) = y'(0)/2, \quad (31)$$

where

$$y'(0) = 2u'(0) = 2z_0. \quad (32)$$

In general

$$y^{(n)}(0) \approx \frac{u^{(n)}(0)}{\sigma(0)} = 2u^{(n)}(0) \ . \quad (33)$$

Thus, the computing algorithm realizing a method of identification of multidimensional Volterra kernels on the basis of ratios (13), (14) and (17) comes down to the decision of the system of linear algebraic equations (24) for each fixed time point of t on an interval [0, T], where T – is modeling time.

For construction operator of the estimation Volterra kernels method of A. N. Tikhonov regularization is used. He on a variation method of creation of the regularized operator is based. This method comes down to finding of an approximate vector of the decision $\bar{z}_\alpha$ which minimizes some functional of smoothing. The only vector meeting a condition of a minimum of the functional of smoothing can be defined from the decision of the system of linear algebraic equations:

$$(A^*A + \alpha I)\bar{z}_\alpha = A^*\bar{u}\,, \quad (33)$$

where A^* – the matrix conjugate to A, I – a unit matrix, α – regularization parameter.

For the choice of value of the α parameter the residual criterion is used [21]:

$$\|Az - \bar{u}\| < \varepsilon, \quad (34)$$

where ε – the set decision error; $\|.\|$ – norm in vector space.

The approximate decision received on the basis of (33) and (34) corresponds to a 0-order of regularization. For increase in smoothness of decisions, the regularized matrix of R is used and solution of the system of linear algebraic equations at value of the parameter α which provides performance of a condition (25) is fended:

$$(A^*A + \alpha R)\bar{z}_\alpha = A^*\bar{u}\,. \quad (35)$$

The regularized matrix of R has tape structure which diagonal elements are equal $r_{ii} = 1-(\Delta a)^{-2}$, and elements in the over diagonal and sub diagonal are equal $r_{ij} = -(\Delta a)^{-2}, i \neq j;\ i, j = \overline{1,L}$ (the first order of regularization) [31].

2.5. Constructing of the Approximation Model. Is developing a method of constructing approximate Volterra model of the NDS [40]. Method identification is based on the approximation $y(t)$ at an arbitrary deterministic signal $x(t)$ in the form of integral power of the polynomial Volterra N-th order (N - order approximation model)

$$\tilde{y}_N(t) = \sum_{n=1}^{N}\hat{y}_n(t) = = \sum_{n=1}^{N}\underbrace{\int_0^t \ldots \int_0^t}_{n\ times} w_n(\tau_1,\ldots,\tau_n)\prod_{i=1}^{n} x(t-\tau_i)d\tau_i\,. \quad (36)$$

Statement 5. Let the input test signals NDS are fed alternately $a_1x(t)$, $a_2x(t)$, ,…, $a_Lx(t)$; a_1, a_2,…,a_L – distinct real numbers satisfying the condition $|a_j| \le 1$ for $\forall j$=1,2,...,L; then

$$\tilde{y}_N[a_jx(t)] = \sum_{n=1}^{N}\hat{y}_n[a_jx(t)] = = \sum_{n=1}^{N}a_j^n\underbrace{\int_0^t \ldots \int_0^t}_{n\ times} w_n(\tau_1,\ldots,\tau_n)\prod_{i=1}^{n} x(t-\tau_i)d\tau_i = = \sum_{n=1}^{N}a_j^n\hat{y}_n(t). \quad (37)$$

The partial components in the approximation model $\hat{y}_n(t)$ are found using the least square method (LSM). This makes it possible to obtain such evaluation in which the sum of squared deviations of responses identified the nonlinear dynamical system $y[a_jx(t)]$ on the model $\hat{y}_N[a_jx(t)]$ response is minimal, i.e., NDS provides a minimum criterion

$$J_N = \sum_{j=1}^{L} \left(y[a_j x(t)] - \tilde{y}_N [a_j x(t)] \right)^2 = \\ = \sum_{j=1}^{L} \left(y_j(t) - \sum_{n=1}^{N} a_j^n \hat{y}_n(t) \right)^2 \to \min, \quad (38)$$

where $y_j(t) = y[a_j x(t)]$. Minimization of the criterion (6) is reduced to solving the system of normal equations of Gauss, which in vector-matrix form can be written as

$$\mathrm{A}'\mathrm{A}\hat{\mathrm{y}} = \mathrm{A}'\,\bar{\mathrm{y}}, \quad (39)$$

where

$$\mathrm{A} = \begin{bmatrix} a_1 & a_1^2 & \cdots & a_1^N \\ a_2 & a_2^2 & \cdots & a_2^N \\ \cdots & \cdots & \cdots & \cdots \\ a_L & a_L^2 & \cdots & a_L^N \end{bmatrix}, \bar{\mathrm{y}} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdots \\ y_L(t) \end{bmatrix}, \quad \hat{\mathrm{y}} = \begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \cdots \\ \hat{y}_N(t) \end{bmatrix}.$$

From (7) we obtain

$$\hat{\mathrm{y}} = (\mathrm{A}'\,\mathrm{A})^{-1}\mathrm{A}'\,\bar{\mathrm{y}} \quad (40)$$

In (8), matrix operations, we obtain

$$\begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \cdots \\ \hat{y}_N(t) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{L} a_j^2 & \sum_{j=1}^{L} a_j^3 & \cdots & \sum_{j=1}^{L} a_j^{N+1} \\ \sum_{j=1}^{L} a_j^3 & \sum_{j=1}^{L} a_j^4 & \cdots & \sum_{j=1}^{L} a_j^{N+2} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=1}^{L} a_j^{N+1} & \sum_{j=1}^{L} a_j^{N+2} & \cdots & \sum_{j=1}^{L} a_j^{2N} \end{bmatrix}^{-1} \times \\ \times \begin{bmatrix} \sum_{j=1}^{L} a_j y_j(t) \\ \sum_{j=1}^{L} a_j^2 y_j(t) \\ \cdots \\ \sum_{j=1}^{L} a_j^N y_j(t) \end{bmatrix}. \quad (41)$$

3. Computer Simulation

Efficiency of the developed methods, algorithms and tools of NDS identification with use of irregular sequences of impulses is confirmed by means of computer simulation in Matlab-Simulink on test object for which analytical expressions for Volterra kernels were received. They were used as a standard at researches of potential accuracy and a noise stability of the developed methods of identification.

3.1. Performance Criterion. For error estimate of experimental determination of Volterra kernels sections is used criterion mean-square error (MSE)

$$\varepsilon = \sqrt{\frac{1}{k} \sum_{t=1}^{k} (w_t - \hat{w}_t)^2}\,, \quad (42)$$

where k – is number of samples at the time slice of measurements, w_t – etalon values of Volterra kernels, $\hat{w}_t$ – estimation value of Volterra kernels received as a result of experimental data (system responses) processing in discrete t time points.

The criterion of the normalized percentage mean-squared error (NPMSE) also is used:

$$\varepsilon_n = 100\% \cdot \sqrt{\frac{\sum_{t=1}^{k} (w_t - \hat{w}_t)^2}{\sum_{t=1}^{k} w_t^2}}\,. \quad (43)$$

3.2. Test Object for Identification. There was chosen an object for the research of identification method which is described by the nonlinear differential equation:

$$\frac{dy(t)}{dt} + \alpha y(t) + \beta y^2(t) = x(t), \quad (44)$$

where α and β are constant real. Structure chart of nonparametric model of the object is illustrated by means of three members of Volterra model in Fig. 5.

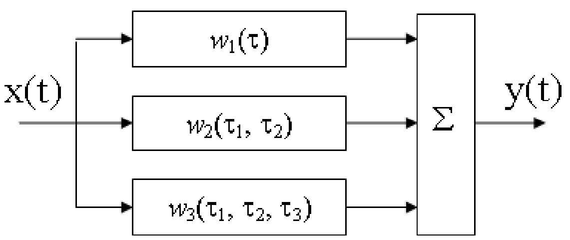


Fig. 5. Structure chart of nonparametric model of the object

Then the Volterra kernels are equal

$$w_1(\tau) = e^{-\alpha\tau}, \quad (45)$$

$$w_2(\tau_1, \tau_2) = \frac{\beta}{\alpha}(e^{-\alpha(\tau_1 + \tau_2)} - e^{-\alpha\tau_2}), \quad \tau_1 \le \tau_2, \quad (46)$$

$$w_3(\tau_1, \tau_2, \tau_3) = \frac{1}{3}\left(\frac{\beta}{\alpha}\right)^2 (e^{\alpha(\tau_1 + \tau_2 - \tau_3)} + +3e^{-\alpha(\tau_1 + \tau_2 + \tau_3)} - \quad (47) \\ -4e^{-\alpha(\tau_2 + \tau_3)} - 2e^{-\alpha(\tau_1 + \tau_3)} + 2e^{-\alpha\tau_3}), \quad \tau_1 \le \tau_2 \le \tau_3.$$

It's considered that $\tau_1 = \tau_2 = \tau_3 = t$, then diagonal Volterra kernels bnsection is received

When $\tau_1 = \tau_2 = \tau_3 = t$ we get the diagonal sections of the Volterra kernels

$$w_1(t,t) = \frac{\beta}{\alpha}(e^{-2\alpha t} - e^{-\alpha t}), \\ w_3(t,t,t) = \left(\frac{\beta}{\alpha}\right)^2 \cdot (e^{-3\alpha t} - 2e^{-2\alpha t} + e^{-\alpha t}). \quad (48)$$

3.3. Wavelet Filtration in Identification Procedure. Procedure of noise smoothing are applied to increase the noise stability of determined identification method to receive estimates of the multidimensional Volterra kernels, based on wavelet transformation [22].

Noise reduction is usually reached by removal of high-frequency components from a the signal range representing an additive mix of information component – received as a result of the Volterra kernels section processing responses and the noise caused by an error of measuring equipment. In relation to wavelet decomposition, it can be realized directly by removal of detailing coefficients of high-frequency levels. Setting some line for their level, and cutting off accordingly detailing coefficients it is possible to achieve reduction of noise level.

For smoothing of identification results when utility from a package of the Wavelet Toolbox expansion of Matlab system with maternal wavelet coif let – coif4 was used at the following values of parameters. Parameter of the rule calculation unit of threshold valuation for restriction of TPTR='minimaxi' coefficients decomposition (by minimax estimation). Parameter of the unit like a threshold of SORH ='s' (flexible) cleaning; the parameter defining a way of SCAL ='one' recalculation threshold (us of a threshold, integrated decomposition for all levels, without rescaling). Depth of data decomposition – 3.

In researches the model of a received noisy assessment of Volterra kernels section is accepted by the additive: $\hat{w}_n(t,\ldots,t)+\xi(t)$ with an even pitch on argument of t, where $\hat{w}_n(t,\ldots,t)$ – is a useful information component, $\xi(t)$ – a hindrance (white Gaussian noise with D dispersion and average zero value).

In Matlab-Simulink MSE assessment were received by means of compensation method of Volterra kernels sections identification diagonal of the second and third orders for NDS test (Fig. 5) at error measurements of responses σ=1, σ=3 and σ=5 % without application and with application of wavelet filtration (Tabl. 1).

Table 1. Mean-square error identification of Volterra kernels of second and third order

Volterra kernels n order	Without application of wavelet filtration			With application of wavelet filtration		
	Error measurements of responses σ = %			Error measurements of responses σ, %		
	1	3	5	1	3	5
2	0,024	0,037	0,045	0,019	0,033	0,037
3	0,025	0,028	0,032	0,014	0,017	0,020

The application of Wavelet filtration in identification procedure on the basis of compensation method allows to receive smoothed estimates of Volterra kernels sections and increase identification accuracy that is criterion of MSE for 20 – 45 %.

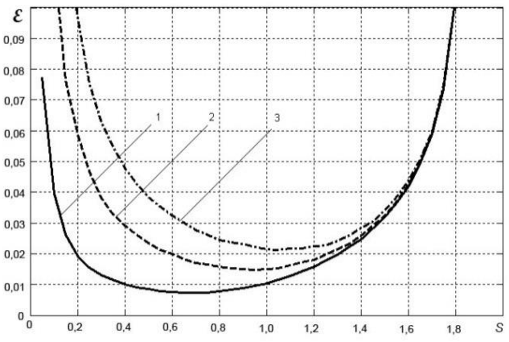
There were received dependences (Fig. 6) of MSE identification by means of interpolation method of diagonal Volterra kernels sections of second (Fig. 6, *a*) and third (Fig. 6, *b*) orders from the area of test S pulse at error of measurement responses σ = 1, σ = 3 and σ = 5 % without application of smoothing procedure of received Volterra kernels sections.

In Fig. 7 and Fig. 8 dependences of the MSE identification results by means of Volterra kernels interpolation method of the second order from the area of test impulses S at error of measurements σ = 1 % are presented. Also after wavelet filtration application to the received estimates of Volterra kernels sections of by means of wavelet transformation on the basis of maternal wavelet coiflet (Fig. 7) with use at various levels of decomposition of L on basis of wavelet coif4 (Fig. 8) are presented. The minimum of MSE identification is reached by using maternal wavelet coiflet – coif4 (Fig. 7) with level of decomposition depth L=4 (Fig. 8). Thus smoothed solutions turn out, and the error of identification decreases bin 1,5 – 2 times.

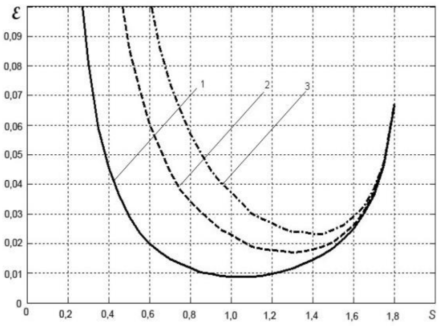
3.4. Comparative Analysis of Identification Methods. The errors arising at application of determined identification methods are investigated, the comparative analysis of their efficiency on the accuracy and noise stability is carried out. The choice of amplitude of impulses sequence is possible to receive optimum estimates on the accuracy of any Volterra section kernels. The procedures of noise reduction based on wavelet transformations are applied to increase the computing stability of identification algorithms.

The NDS (Fig. 5) received by means of three methods of determined identification are given in Tabl. 2 – compensation, approximating and interpolation methods by NPMSE of diagonal Volterra kernels sections assessment of second order for test, at an error of responses σ = 1, σ = 3 and σ = 5 % measurements without application and with wavelet filtration application.

Dependence diagram of MSE Volterra kernels identification of second order from the area S (amplitude) trial impulses in the conditions of ideal experiment (exact measurements) and taking into account errors of measurement responses (error σ = 3 %) are submitted in Fig. 9 and Fig.10 respectively.



a



b

Fig. 6. Mean-square error identification of Volterra kernels of second and third order dependences of identification by Volterra kernels interpolation method of the second (*a*) and the third orders(*b*) from the area of *S* test pulses respectively for *r*1=*r*2=1, and *r*1=*r*2=2 at errors of measurements: *1* – 1 %; *2* –3 %; *3* – 5 %

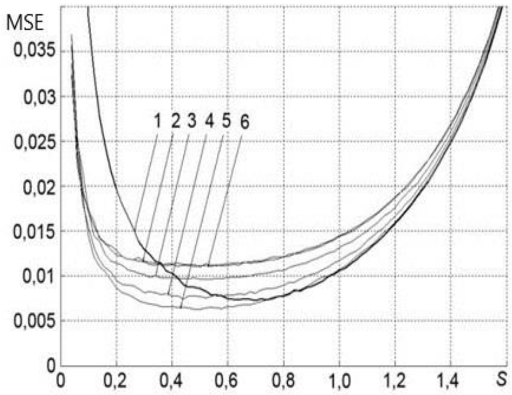


Fig. 7. Dependences of the MSE identification of Volterra kernel second order on the area of test impulses *S* at error of measurements of 1 %:
1 – without filtration; *2* – at wavelet-filtration application with help wavelet coif1; *3* – coif2; *4* – coif3; *5* – coif4; *6* – coif5

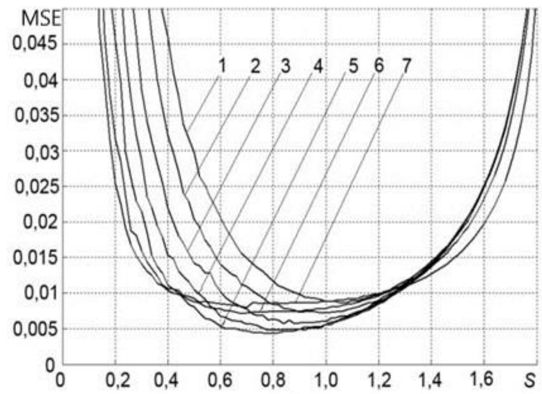


Fig. 8. Dependences of the MSE identification of Volterra kernel second order on the area of test impulses *S* at error of measurements of 1%:
1 – without filtration; *2* – at wavelet-filtration application on a basis of wavelet coif4 with levels of decomposition of *L* = 1; *3* – *L* = 2; *4* – *L* = 3; *5* – *L*=4; *6* –*L*=5; *7* – *L*=6

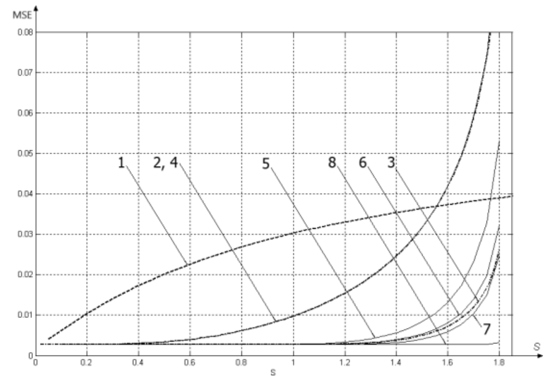


Fig. 9. Dependence diagrams of MSE assessment of diagonal Volterra kernel section of second order from the area *S* trial impulses at identification on exact measurements:
1 – for a compensation method; *2* – at *r*1=*r*2=1; *3* – at *r*1=*r*2=2 for an interpolation method; *4* – at *N*=2 and *N*=3; *5* – at *N*=4 and *N* =5, 6 – at *N*=6; *7* – at *N*=7; *8* – at *N*=8 for an approximation method

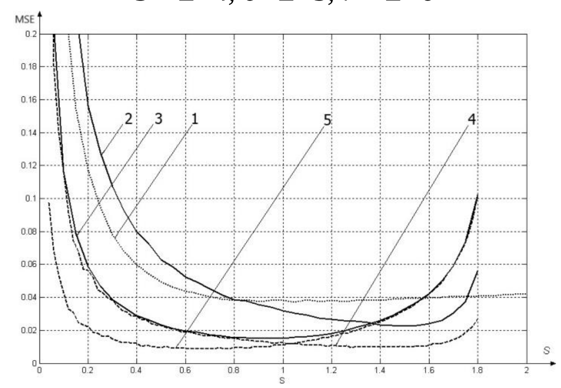


Fig. 10. Dependence diagrams of MSE assessment of diagonal Volterra kernel section of second order from the area *S* trial impulses at identification with error 3 %:
1 – for compensation method; *2* – at *N*=2; *3* – at *N*=4 and *N*=5 for approximation method; *4* – at *r*1=*r*2=1; *5* – at *r*1=*r*2=2 for interpolation method

The results of the estimated of Volterra kernels with basis of identification methods on based compensation (Fig. 11), of approximation (Fig. 12) and of interpolation (Fig. 13). Estimations of diagonal Volterra kernels sections of second and third orders for test NDS at error of measurements responses by σ = 1 % without application and with wavelet filtration application on basis of maternal wavelet coif4 with decomposition level L=4 received.

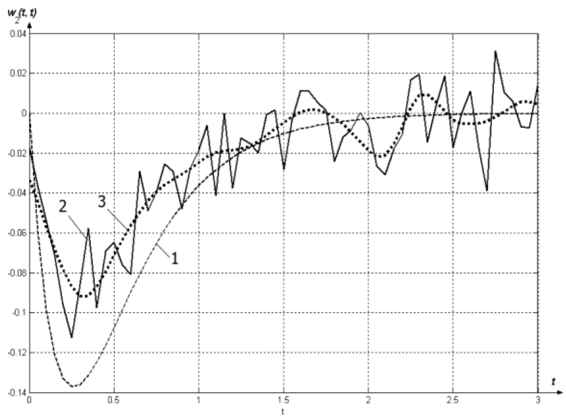
The analysis of identification results of three methods by means of pulse sequences (Fig. 11, Fig. 12 and Fig. 13) on test object (Fig. 5) shows that the highest precision and noise stability possesses the interpolation method of identification, consisting in responses differentiation on parameter amplitude of trial impulses (17). Least exact of considered methods of determined identification is compensatory method (4), (5).

Researches of a robust method were implement by means of computer modeling in the environment of Matlab–Simulink at the following values of parameters of test impulses signals: $\Delta\tau$=0,01, $a_{\max}$=100.

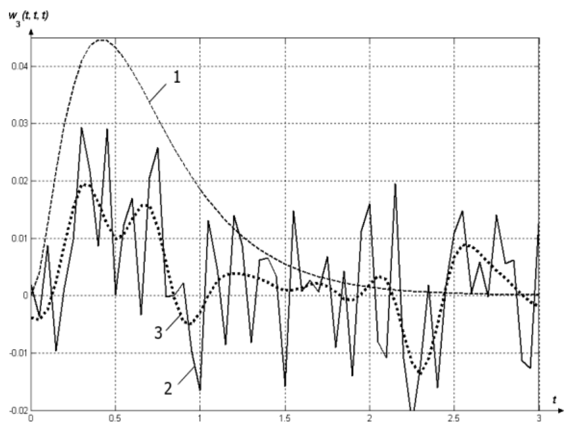
On the Fig. 14 results of identification of a test object (Fig. 5) on basis with accurate response measurements of responses and without application of regularization – estimates of Volterra kernels of the first order of $w_1(t)$ (Fig. 14, *a*) and the diagonal section of Volterra kernels of the second order of $w_2(t,t)$ (Fig. 14, *b*) are presented. Experiments were carried out with a step on amplitude of test impulses Δa, successively taking values from a set {8, 10, 16, 20 and 40}. The number of experiments L is 8, 10, 16, 20, and 40, respect*ively*. The best results of identification are received at Δa=5 (L=40).

Estimates of diagonal section of Volterra kernels of the second order of $w_2(t,t)$ at an error of measurements of 1 % on the basis of the decision of SLAE (35) for Δa=4 (L=50) without regularization are presented in Fig. 15, *a*. The big mistakes received at the same time are not acceptable in practice, NPMSE makes 244,2 %. In Fig. 15, *b* estimates are given $w_2(t,t)$, received by means of a method of regularization and smoothings with use of Wavelet-transformation [22, 34]. At this NPMSE of identification makes 2,95 %, respectively accuracy increased by 82,8 times.

For test NDS (Fig. 5) are received the results of identification by means of four of the computational methods – compensation method, approximation method, interpolation method and robust method are given in Tabl. 2. Here are values of the criterion NPMSE obtained with using the methods deterministic identification, at estimation of the diagonal section of a second order Volterra kernel from measurements of responses with an error σ=1, σ=3 and σ=5 % without application and with Wavelet-filtration application.

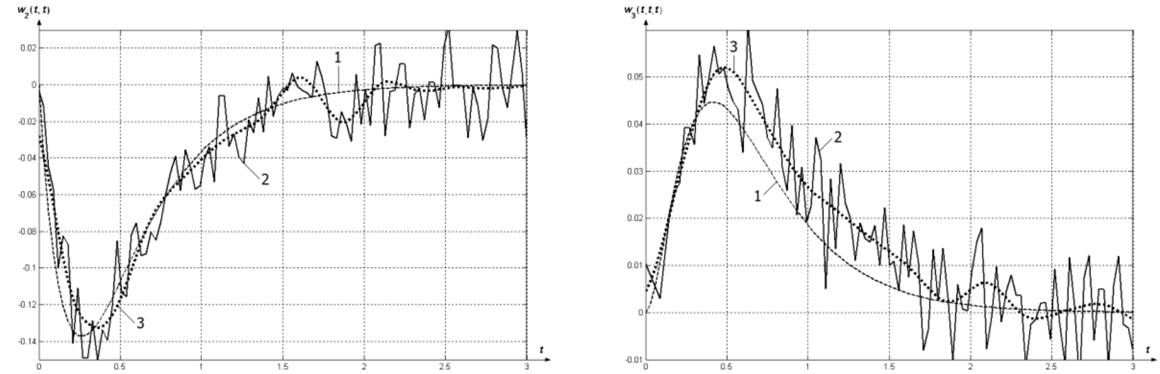


a



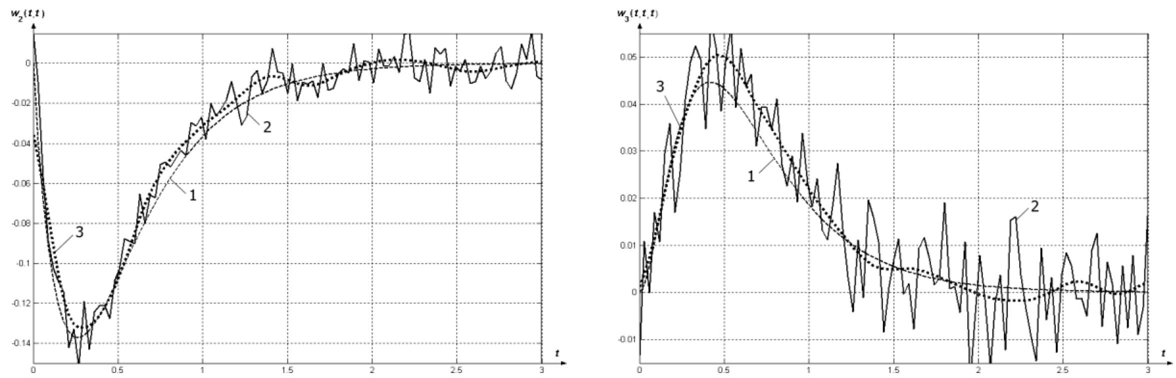
b

Fig. 11. Result of diagonal identification Volterra kernel sections of the second (*a*) and the third (*b*) NDS orders by means of compensation method at measurement error of 1 %:
1 – etalon of the Volterra kernel; *2* – identified kernel; *3* –identified kernel at wavelet-filtration application on wavelet basis coif4 with level of decomposition L=4



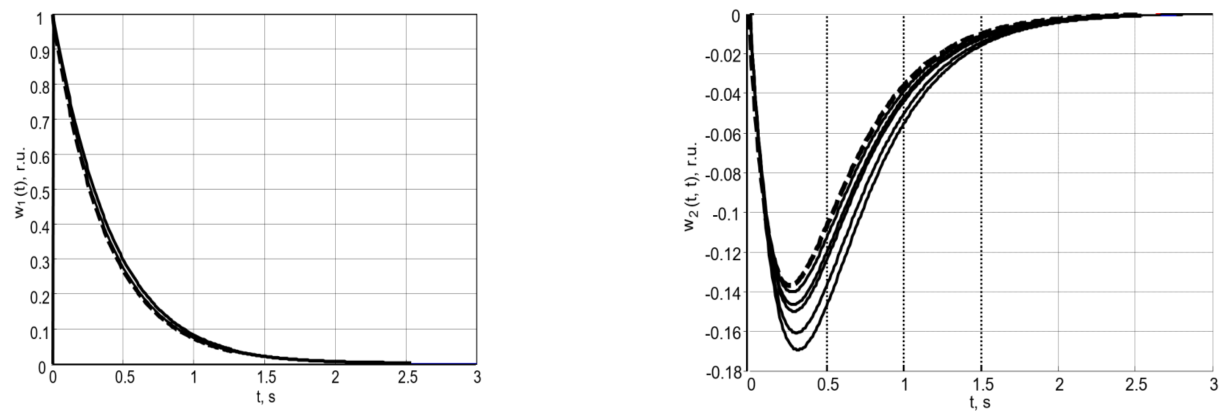
a *b*

Fig. 12. Result of diagonal identification Volterra kernel sections of the second (*a*) and the third (*b*) NDS orders by means of approximation method (N=4) at measurement error of 1 %:
1 – etalon of the Volterra kernel; *2* – identified kernel; *3* – identified kernel at wavelet filtration application on wavelet basis coif4 with level of decomposition L=4



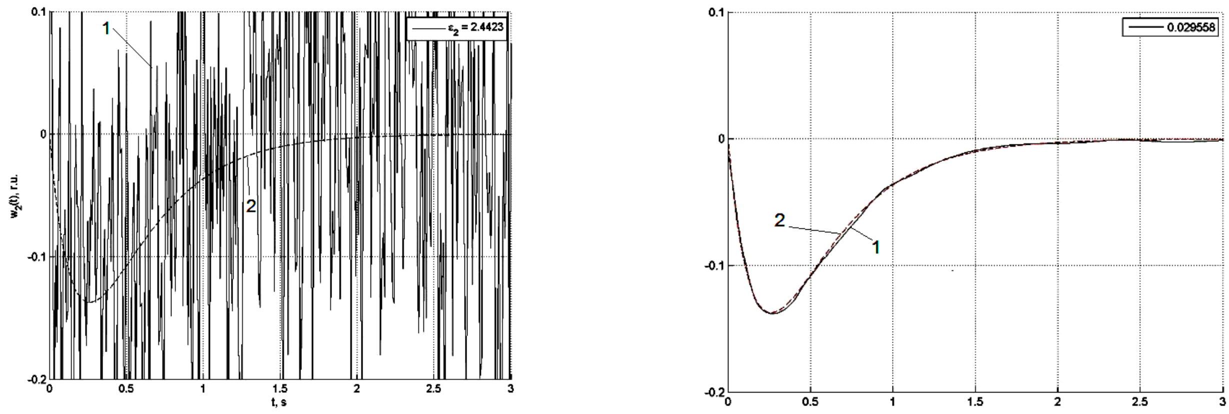
a *b*

Fig. 13. Result of diagonal identification Volterra kernel sections of the second (*a*) and the third (*b*) NDS orders by means of interpolation method (r_1+r_2=4) at measurement error of 1 %:
1 – etalon of the Volterra kernel; *2* – identified kernel; *3* – identified kernel at wavelet filtration application on wavelet basis coif4 with level of decomposition L=4



a *b*

Fig. 14. Results of identification of a test object without regularization at exact measurements of responses: estimates of Volterra kernel of the first order (*a*) and diagonal section of Volterra kernel of the second order (*b*). A dotted line – etalon



a *b*

Fig. 15. Estimates of diagonal section of NDS of the second order of a test object of $w_2(t,t)$, at errors of measurements of 1 % on the basis of the decision of SLAE (24) without regularization (*a*) and the regularizations received by means of a method and smoothing with wavelet-filtration use (a robust method) at Δa=4, L=50 (*b*): *1* – result of identification; *2* – etalon

Table 2. Normal Percentage Mean-square error identification of the Volterra kernel second order

Parameters of methods	Quantity experiments	Quantity operations	Minimum NPMSE (%) at an error of measurements σ (%)					
			Without application Wavelet – filtrations			With application Wavelet – filtrations		
			σ=1%	σ=3%	σ=5%	σ=1%	σ =3%	*σ=5%*
	Compensation method							
	2	4	44,0	66,5	77,1	30,1	43,7	53,7
N	Approximation method							
2	2	4	12,6	25,9	37,0	10,8	15,0	18,3
3	3	6	11,9	24,5	33,5	9,08	13,3	16,9
4	4	8	15,7	40,3	63,3	11,2	18,1	24,5
5	5	10	15,2	38,0	58,7	11,1	17,0	22,7
6	6	12	18,7	50,4	80,5	11,9	20,5	29,3
$r_1=r_2$	Interpolation method							
1	2	5	13,0	26,3	37,5	10,9	15,5	19,2
2	4	9	14,7	36,5	58,1	11,2	16,8	23,6
3	6	11	19,6	54,1	88,1	11,6	20,8	31,5
4	8	12	25,6	77,3	126,0	13,1	25,1	44,0
Δa	Robust method							
4	50	–	6,8	18,4	–	3,0	5,8	–

Results of comparison of a regularized method of identification on the basis of the decision of SLAE (35) and an interpolation method where for numerical differentiation formulas in final differences are used and natural regularization – optimization of a step on amplitude of test impulses is applied, for a kernel of the second order are provided in Tab. 3.

4. Identification Technique of the “Black Box”

The technique of Volterra kernels identification is developed for systems of unknown structure (like “Black Box”).

1. The greatest possible amplitude of test impulses at which identified NDS is still steady (on limit of stability) is set. Duration of impulses $\Delta\tau$ gets out of a condition:

$$\Delta\tau \le 0{,}05\frac{\tau_{\min}}{n}, \qquad (49)$$

where: $\tau_{\min}$ – the minimum constant of time of linear system part, n – an order of defined Volterra kernels.

2. Procedure of NDS identification is consistently applied at various values of a test impulses amplitude and

$$a_{i+1} = \mu a_i,\ 0 < \mu < 1,\ i = 0,1,2,\ldots. \qquad (50)$$

For each pilot study of identified system and processing of received responses according to one of algorithms of identification (4), (5) or (11), (13), (14), or (17), since the second identification experiment, there is a mean square deviation of ε between the next results of n order Volterra kernels estimates.

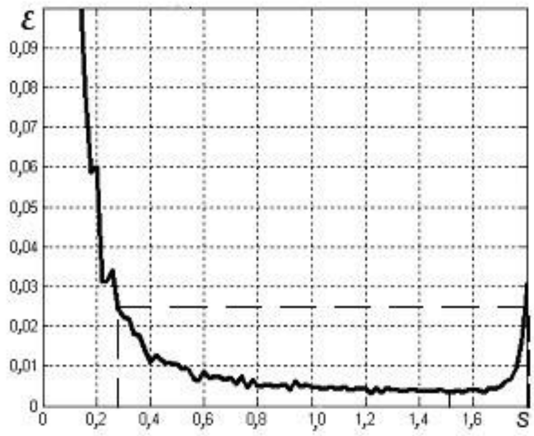
3. On basis of these results there is quasi-optimal amplitude of test pulse signals with which next results of n order Volterra kernels $\hat{w}_{a_{i+1}}, \hat{w}_{a_i}$ identification will be the closest that is criterion of a mean square deviation. The decision (estimated value is $\hat{w}_{a_i}$) gets out at value of the amplitude a_i under condition

$$\left\| \hat{w}_{a_{i+1}} - \hat{w}_{a_i} \right\| \to \min_{a_i}. \qquad (51)$$

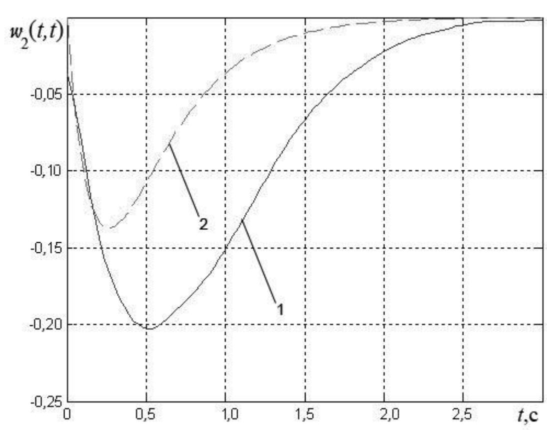
The technique is approved on a problem of test object identification (Fig. 5) considered as NDS with unknown structure. The received results are presented in Fig. 16. The dependence diagram of MSE identification is given in (Fig. 16, *a*) from the amplitude of test impulses on which values of amplitude are marked a=90 (S=1,8) and a=14 (S=0,28) at which the mean square deviation of identification results accept identical values (ε=0,024) are noted. Results of Volterra kernels identification of second order corresponding to them, received by interpolation method at r_1=r_2=2, are given in (Fig. 16, *b*) and (Fig. 16, *c*). The result of identification corresponding to the minimum value of a mean square deviation at ε=0,004 is given in (Fig. 16, *d*). Thus optimum amplitude of impulses of is a=76 (S=1,52).

Table 3. Comparative analysis of identification methods on the example of estimation Volterra kernel a second order of NDS

Identification method	Minimum NPMSE, ε_n, % at an error of measurements σ, %			
	Regularization application		Application Wavelet-filtration	
	σ=1	σ=3	σ=1	σ=3
Identification on the basis of the decision of SLAE (35), L=50	6,7	18,4	2,83	5,85
Identification on the basis of an interpolation method	13,0	26,3	10,9	15,5



a



b

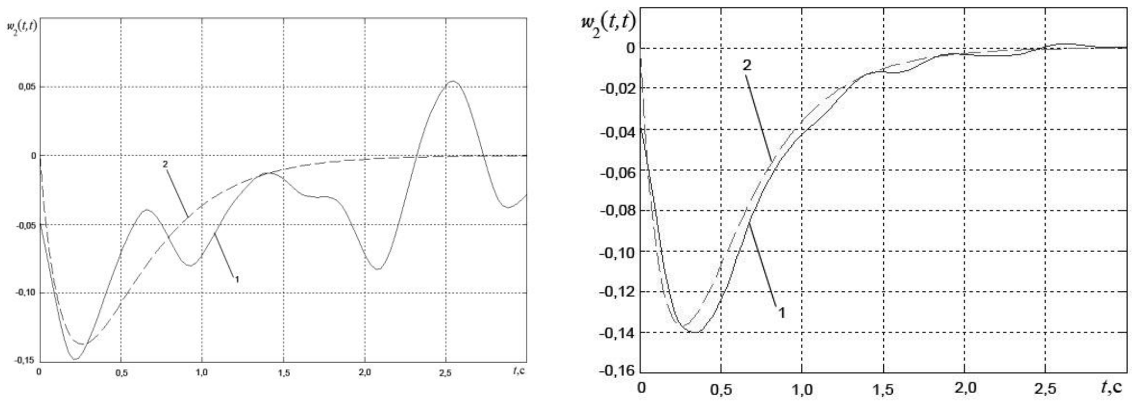


Fig. 16. Result of diagonal identification Volterra kernel sections of the second order NDS with unknown structure with using interpolation method at $r_1 = r_2 = 2$: MSE identification (*a*) and with various means of amplitude test impulses:
S=1,8 (*b*), 0,28 (*c*) and 1,52 (*d*) respectively; *1*– result of identification; *2* – etalon

Conclusion

Methodological and algorithmic bases of creation of information models of continuous subjects to control in the form of Volterra kernels on the basis of data of an experiment an entrance exit are developed.

The statements proving methods of the determined identification of nonlinear dynamic systems with one entrance and an exit and also for systems with many entrances and many exits on the basis of Volterra models in a time domain – with use as test signals of the irregular sequences of impulses are proved. Advantage of the considered methods of the determined identification – compensation, approximation, interpolation and robust in comparison with methods of statistical identification, are comparative simplicity of generation of test signals and simplicity of processing of empirical data.

It is shown that estimates, optimum on accuracy, any diagonal and on diagonal sections of Volterra kernels are the choice of parameters of the sequence of impulses – duration, amplitude and an interval between impulses.

The analysis of errors of a compensation method of identification – the methodical, caused uncompensated processing by a contribution in a response of a system of members of Volterra serie whose order is higher than an order of the estimated Volterra kernels measurements of responses is made. It is shown that at reduction of amplitude of trial impulses the methodical error decreases, but at the same time the relative error of measurements increases.

The new interpolation method of identification of the NDS in the form of Volterra series based on allocation of a partial component of Volterra series – the whole uniform regular functionality of Volterra *n*-th is offered degree, by means of *n*-fold differentiation of responses of the NDS in the parameter amplitude of test influences, the corresponding computing algorithms realizing an identification method are developed. The method allows to mi NDS caused by influence of partial components of a response above *n*-th. However, implementation of a method of identification leads to errors of assessment of the Volterra kernels, which level depends on amplitude of test signals and the accuracy of measurements of responses.

It is shown that the known amplitudes of test signals for use in an approximating method of identification which is based on drawing up linear combinations of responses of the NDS for test influences with different amplitude are not optimum and the choice of amplitudes of test influences and the corresponding weight coefficients providing the minimum error of assessment of multidimensional Volterra kernels of the identified system is proved.

A new robust method of deterministic identification of non-linear dynamic systems on the basis of model Volterra in time domain, for the numerical realization of which can be used unlimited top number of experiments with the “input-output”, and the application of the method of regularization, the processing of noisy experimental data allows to increase the accuracy and noise immunity of the procedure for identification. Set the effectiveness of the developed methods and appropriate tools for their introduction in the practice of diagnostic studies of technical and biological objects in the industry and scientific research organizations.

The methods of deterministic identification considered here were the basis for the creation of intellectual information technology for diagnosing complex continuous objects of different physical nature [41-46]. This the model-based information technology of diagnosis was effectively used to construct a classifier of the states of the Switched-Reluctance Electric Motors [42]. This methods of the nonlinear dynamical systems identification using the Volterra model in the frequency domain are developed [47-54]. Identification methods apply for simulation of wireless communication channels using Volterra model in frequency domain [55 – 61]. Also, identification methods for building of nonlinear dynamic model oculo-motor system human based on Volterra kernels apply [62 – 69]. The information model of the photosynthetic reaction center in the form of Volterra kernels of the first, second and third orders was constructed on the basis of deterministic identification methods [70].

Appendix A. Prove of the Statement 1 и 2

The model of the test signal, which is an irregular sequence consisting of no more than m pulses of the same amplitude $S_1=...=S_m=S$ acting at times τ_i can be written as

$$x(t)=\sum_{i=1}^{m}\theta_i S\,\delta(t-\tau_i), \qquad (A.1)$$

where θ_i – parameter representing amount of impulses and time delays between them within the test impulse sequence – in case $\theta_i=1$, impulse is present in sequence at time moment τ_i, in case $\theta_i=0$ – impulse is not present.

After substituting (1) into (A1), we get the NDS response as

$$y(t,\theta_1,...,\theta_m)=\sum_{n=1}^{\infty}S^n\sum_{j_1,...,j_n=1}^{m}\prod_{k=1}^{n}\theta_{j_k}\,w_n(t-\tau_{j_1},...,t-\tau_{j_n}). \qquad (A.2)$$

After substituting expression (A.2) to (4)

$$\hat{w}_m(t-\tau_1,...,t-\tau_m)=\frac{(-1)^m}{m!}\sum_{n=1}^{\infty}S^{n-m}\sum_{\theta_1,...,\theta_m=0}^{1}\sum_{j_1,...,j_n=1}^{m}(-1)^{\sum_{i=1}^{m}\theta_i}\times \qquad (A.3)$$
$$\times\prod_{k=1}^{n}\theta_{j_k}\,w_n(t-\tau_{j_1},...,t-\tau_{j_n}).$$

Sum of n elements in expression (A.3) might be represented as

$$\sum_{n=1}^{\infty}\sigma_n=\sum_{n=1}^{m-1}\sigma_n+\sigma_m+\sum_{n=m+1}^{\infty}\sigma_n, \qquad (A.4)$$

where

$$\sigma_n=S^{n-m}\sum_{\theta_1,...,\theta_m=0}^{1}\sum_{j_1,...,j_n=1}^{m}(-1)^{\sum_{i=1}^{m}\theta_i}\times \qquad (A.5)$$
$$\times\prod_{k=1}^{n}\theta_{j_k}\,w_n(t-\tau_{j_1},...,t-\tau_{j_n}),\ \text{where } n\neq m;$$

$$\sigma_m=\frac{(-1)^m}{m!}\sum_{\theta_1,...,\theta_m=0}^{1}(-1)^{\sum_{i=1}^{m}\theta_i}\times\sum_{j_1,...,j_m=1}^{m}\prod_{k=1}^{m}\theta_{j_k}\times \qquad (A.6)$$
$$\times w_m(t-\tau_{j_1},...,t-\tau_{j_m}),\ \text{where } n=m.$$

In (A.6) amount by $j_1,\dots,j_m$ might be presented as

$$\sum_{j_1,...,j_m=1}^{m}=\sum_{\substack{j_1,...,j_m=1\\ j_1\neq...\neq j_m}}^{m}{}'+\sum_{\substack{j_1,...,j_m=1\\ j_p=j_q}}^{m}{}'', \qquad (A.7)$$

where p and q take values from a subset $\overline{1,m}$, $p\neq q$.

At (A.6) as the result of sum based on $\theta_1,...,\theta_m$, applied to the first operand of sum $j_1,\dots,j_m$ where $j_1\neq...\neq j_m$ (A.7), taking into account that Volterra kernels are symmetric functions, which means $w_m(t-\tau_1,...,t-\tau_m)$, so differed by the arguments order are identically even

$$\sigma'_m=\frac{(-1)^{2m}}{m!}m!\,w_m(t-\tau_1,...,t-\tau_m)= \qquad (A.8)$$
$$=w_m(t-\tau_1,...,t-\tau_m).$$

At (A.6) as the sum result based on $\theta_1,...,\theta_m$, applied to the second operand of summarization $j_1,\dots,j_m$ where $j_p=j_q$

$$\sigma''_m=\frac{(-1)^m}{m!}\sum_{\theta_1,...,\theta_m=0}^{1}(-1)^{\sum_{i=1}^{m}\theta_i}\sum_{\substack{j_1,...,j_m=1\\ j_p=j_q}}^{m}{}''\prod_{k=1}^{m}\theta_{j_k}\times \qquad (A.9)$$
$$\times w_m(t-\tau_{j_1},...,t-\tau_{j_m})=0.$$

For equality proving (A.9) sum on $\theta_1,...,\theta_m$ might be represented as

$$\sum_{\theta_{k_1},...,\theta_{k_m}=0}^{1}(-1)^{\sum_{g=1}^{r}\theta_{k_g}}\sum_{\theta_{l_1},...,\theta_{l_s}=0}^{1}(-1)^{\sum_{h=1}^{s}\theta_{l_h}}\prod_{h=1}^{s}\theta_{l_h}, \qquad (A.10)$$

where $\theta_{k_1},...,\theta_{k_r}$ – elements from set $\{\theta_1,...,\theta_m\}$, which are not members of product $\prod_{h=1}^{s}\theta_{l_h}$; $r+s=m, k_1\neq\ldots\neq k_r\neq l_1\neq\ldots\neq l_s\in\overline{1,m}$.

As far as

$$\sum_{\theta_1,...,\theta_n=0}^{1}(-1)^{\sum_{i=1}^{n}\theta_i}=0, \qquad (A.11)$$

which can be proven with the mathematical induction method, то $\sigma''_m=0$ и $\sigma_m=\sigma'_m$.

Then from (A.8) it appears that

$$\sigma_m=w_m(t-\tau_1,...,t-\tau_m). \qquad (A.12)$$

It can be shown that first member in a sum (A.4)

$$\sum_{n=1}^{m-1}\sigma_n=0. \qquad (A.13)$$

For all $n<m$

$$\sigma_n = S^{n-m} \sum_{\theta_1,\ldots,\theta_m=0}^{1} \sum_{j_1,\ldots,j_n=1}^{m} (-1)^{\sum_{i=1}^{m}\theta_i} \times$$
$$\times \prod_{k=1}^{n} \theta_{j_k} w_n(t-\tau_{j_1},\ldots,t-\tau_{j_n}) = 0. \quad (A.14)$$

Since $\prod_{k=1}^{n}\theta_{j_k}$ consists of elements, which are a subset from a set $\{\theta_1,\ldots,\theta_m\}$ which means that equality (A.14) might be proven in the same manner as (A.9).

Third member in (A.4) is not equal to zero so in its place adds the error for Volterra kernels definition. In case square S of impulses in a test sequence will be taken small enough, then error $\Delta(S)$ while Volterra kernels identification for m-th member based on σ_n where $n>m$ (A.4) is proportional to S^{m+1}, and appears to be (m-1)-th order.

Which means

$$\hat{w}_m(t-\tau_1,\ldots,t-\tau_m) = $$
$$= w_m(t-\tau_1,\ldots,t-\tau_m) + O(S^{m+1}). \quad (A.15)$$

Statement 1 is proven. Statement 2 can be proven in the same manner.

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Received 27.11.2018

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МЕТОДИ ДЕТЕРМІНОВАНОЇ ІДЕНТИФІКАЦІЇ НЕЛІНІЙНИХ ДИНАМІЧНИХ СИСТЕМ НА ОСНОВІ МОДЕЛІ ВОЛЬТЕРРА

Анотація. *Досліджуються методи детермінованої ідентифікації нелінійних динамічних систем на основі моделей Вольтерра у часовій області: компенсаційний, апроксимаційний, інтерполяційний і робастний. В якості тестових впливів використовуються неперіодичні імпульсні послідовності. Обґрунтовуються обчислювальні методи ідентифікації у вигляді ядер Вольтерра для одно– і багатовимірних систем. Запропоновано методику ідентифікації систем з невідомою структурою в умовах реального експерименту. Досліджуються похибки, що виникають при застосуванні розглянутих методів ідентифікації. Наведено порівняльний аналіз їх ефективності по точності і обчислювальної стійкості. Показано, що при виборі відповідних параметрів імпульсної послідовності, таких як тривалість, амплітуда та інтервал часу між імпульсами, можна з максимально досяжною точністю знайти перетини ядер Вольтерра. Для підвищення обчислювальної стійкості алгоритмів ідентифікації застосовуються процедури шумозаглушення, що засновані на вейвлет-перетворенні.*

Ключові слова: *нелінійні динамічні системи; ідентифікація; модель Вольтерра; ядра Вольтерра; вейвлет-перетворення*

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МЕТОДЫ ДЕТЕРМИНИРОВАННОЙ ИДЕНТИФИКАЦИИ НЕЛИНЕЙНЫХ ДИНАМИЧЕСКИХ СИСТЕМ НА ОСНОВЕ МОДЕЛИ ВОЛЬТЕРРА

Аннотация. *Исследуются методы детерминированной идентификации нелинейных динамических систем на основе моделей Вольтерра во временной области: компенсационный, аппроксимационный, интерполяционный и робастный. В качестве тестовых воздействий используются непериодические импульсные последовательности. Обосновываются вычислительные методы идентификации в виде ядер Вольтерра для одно– и многомерных систем. Предложена методика идентификации систем с неизвестной структурой в условиях реального эксперимента. Исследуются погрешности, возникающие при применении рассмотренных методов идентификации. Приведен сравнительный анализ их эффективности по точности и вычислительной устойчивости. Показано, что при выборе соответствующих параметров импульсной последовательности, таких как длительность, амплитуда и интервал времени между импульсами, можно с максимально достижимой точностью найти сечения ядер Вольтерра. Для повышения вычислительной устойчивости алгоритмов идентификации применяются процедуры шумоподавления, основанные на вейвлет-преобразовании.*

Ключевые слова: *нелинейные динамические системы; идентификация; модель Вольтерра; ядра Вольтерра; вейвлет-преобразование*